

Background

In class, we saw how to model “real world scenarios” (word problems) as exponential functions, and used properties of exponents and logarithms to solve for missing variables. In the end, everything ended up coming back to the following equation:

$$A = Pe^{rt}$$

where A is the new amount, P is the initial population (or principal investment, in compound interest terms), e is the constant $e \approx 2.718$, r is the **growth constant** (sometimes written as k , if you’re using online resources to assist you), and t is the time (sensitive to units - make sure to read the problem to figure out whether you’re dealing with seconds or years!).

Your Task

Answer the following questions:

1. The radioactive isotope Carbon-14 has a half-life of about 5730 years. If you start with 200g of Carbon-14, how much will be left after 1000 years?
2. The radioactive isotope Aluminum-26 has a half-life of 717000 years, and decays into the stable nuclide Magnesium-26 (don’t worry about the term “stable nuclide”).

You have recently discovered an ancient tomb, and like any sensible young person you immediately jumped at the chance for an adventure. Deep inside the tomb, you find an inscription that translates roughly to “May this sphere of purest aluminum guide the fallen to the afterlife. With this, we seal the tomb.”. You carefully pick up the sphere, and then narrowly escape the rolling boulder and series of spike traps triggered by the pressure switch.

Back in the lab, you determine that 11% of the sphere consists of Magnesium-26. How long ago was the tomb sealed?

3. You have a sample of 2kg of the radioactive isotope Sekoridium-673. After 7 years, you have **lost** 1.2kg of the sample to decay. What is the half-life of Sekoridium-673? Write the **exact** answer, and also provide an estimate to the nearest tenth of a second (use a calculator for this part).
4. A population of bacteria grows according to the following formula:

$$A = 300e^{.34t}$$

where t is measured in seconds.

- (a) How many bacteria are there initially?
 - (b) How long will it take for the population of bacteria to double? Write the **exact** answer, and also provide an estimate to the nearest tenth of a second (use a calculator for this part).
 - (c) You would like to harvest this bacteria for use in yoghurt. You want to establish a routine such that you harvest all but 300 of the bacteria every d seconds. For example, if there are 1073 bacteria at the time of harvest, you’ll harvest 773 of them. You would like to maximize the number of bacteria per second (BPS) you can harvest (on average - so if you harvest 24 bacteria every 6 seconds, your BPS would be $24/6 = 4$).
True or False: Choosing a larger value for d (the interval in between harvests) will result in a larger BPS. If true, briefly explain why. If false, give a counterexample (a choice of two interval lengths $d_1 < d_2$ such that your BPS is as high or higher with d_1 than with d_2).
5. Find an example of something in the real world (in nature, science, business, etc.) that grows or decays exponentially, other than the examples I’ve shown in class (compound interest, organism population size, and radioactive isotope decay). Write an equation in $A = Pe^{rt}$ form to describe the growth or decay, and briefly explain how the equation you wrote corresponds to the real-world thing.